Universidad de Puerto Rico
Departamento de Matemáticas
MATE 3152 – Examen Final– 16 de mayo de 2012

Apellidos: ____________________ ____________________ Nombre ____________________

No. Estudiante: ______________________________ Profesor: V. Keyantuo Sección 002

Para obtener crédito muestre todo su trabajo. Explique claramente su contestación.
No credit will be given for unjustified answers.

Part I (20 pts)

(1) (6 pts) Polar equation for the line passing through $B(24, 24)$ and perpendicular to the line $x - y = 16$.

(2) (6 pts) Compute $\dfrac{d}{dx}\tanh^{-1}(\cos x)$

(3) (4 pts) $\dfrac{d}{dx}\ln(x + \sqrt{x^2 + 1})$

(4) (4 pts) Polar equation for the circle through the origin and centered at $C(40, 0)$.

Part II (25 pts)

Compute the integrals:

(1) (5 pts) $\displaystyle\int \frac{12x^7}{2+x^8}dx$

(2) (5 pts) $\displaystyle\int \frac{120(\cos^3 x)\sin x}{(16+\cos^4 x)^2}dx$

(3) (5 pts) $\displaystyle\int \frac{-8x-20}{x^2+10x+25}dx$

(4) (5 pts) $\int e^{-\sqrt{x}}dx$

(5) (5 pts) $\int e^{x} \sin x \ dx$

Part III (10 pts)

(1) (10 pts) Solve the differential equation $y'+y = \sin x$ given that $y(0) = \pi$. (*Hint. see previous problem*)

Part IV (10 pts)

Evaluate the following limits:

(1) (6 pts) $\lim_{x\to 0} \left[\frac{12}{\sin x} - \frac{12}{x} \right]$

(2) (4 pts) $\lim_{x\to\infty} \frac{e^x}{x^2}$

Part V (16 pts)

Say whether the following series converge or not (no credit for unjustified answers).

(1) (8 pts) $\sum_{n=1}^{\infty} \ln(1 - \frac{\pi}{n^4})$

(2) (8 pts) $\sum_{n=1}^{\infty} \frac{(-1)^n \pi}{(n+1)(n+2)}$

Part VI (10 pts)

Obtain the Taylor series about $x = c$ and specify the radius of convergence.

(1) (10 pts) $f(x) = \log_3(4 + x)$, $c = 5$. (*Hint. Compute* $f'(x)$ *and observe that* $f(5) = 2$.)

Part VII (11 pts)

(1) (a) (4 pts) Sketch the graph represented in polar coordinates by $r = 16\cos(\theta - \frac{\pi}{4}.)$

(b) (7 pts) Use polar coordinates to compute the area enclosed by the above graph. Use polar coordinates to compute the area (*no credit if another method is used*).

Part IX (8 pts)

(1) (8 pts) Find the length of the curve given parametrically by: $\begin{cases} x = 5t \\ y = \cosh(5t) \end{cases}$ where $0 \leq t \leq 12$.